ECON 251: Intermediate Microeconomics
Department of Economics, Lafayette College Spring 2024
Instructor: Prof. Sayorn Chin (he/him/his)

Problem Set 6

Due: 27 Mar (Physical Copy in Class)

Instruction

Work in teams of 2-3 people, provide your responses typed in LaTeX. Handwritten submissions will not be accepted. The primary purpose of the problem sets is to give you experience thinking and working through economic problems. Getting the right answer is much less important than understanding the right answer and how it was derived. Accordingly, problem sets are graded on a combination of effort and accuracy. Your investment or lack of investment in these assignments will determine your success in the course as problem set investment is strongly correlated with exam performance.

1. (1 point) Explain why the concepts of constant, increasing, and decreasing returns to scale make sense when applied to isoquants, but would not make sense in the theory of the consumer if applied to indifferent curves. That is, why does the spacing between successive isoquants make sense, while the spacing of successive indifference curves does not?

2. (3 points) Suppose a firm's production function is $y = x_1^{\frac{1}{4}} x_2^{\frac{1}{4}}$. The prices of the inputs are $w_1 = 1$ and $w_2 = 2$.

    a. (1 point) Show that the long-run conditional factor demands are $x_1^*(y) = 2^{\frac{1}{2}} y^2$ and $x_2^*(y) = \frac{y^2}{2^{\frac{1}{2}}}$.

    b. (1 point) Show that the long-run cost function is $C(y) = 2\sqrt{2}y^2$.

    c. (1 point) Show that the long-run supply curve for the firm is given by $y^*(p) = \frac{p}{4\sqrt{2}}$.

3. (6 points) A firm produces computers with two factors of production: labor L and capital K. Its production function is $y = \frac{LK}{10}$. Suppose the factor prices are $w_L = 10$ and $w_K = 100$.

    a. (2 points) Graph the isoquants for y equals to 1, 2, and 3. Does this technology show increasing, constant, or decreasing returns to scale? Why?

    b. (1 point) Derive the conditional factor demands.

    c. (1 point) Derive the long-run cost function $C(y)$.

    d. (2 points) If the firm wants to produce one computer, how many units of labor and how many units of capital should it use? How much will it cost? What if the firm wants to produce two computers?

e. (2 points) Derive the firm's long-run average cost function $AC(y)$ and long-run marginal cost function $MC(y)$. Graph $AC(y)$ and $MC(y)$. What is the firm's long-run supply curve?

4. (5 points) Consider a production function which uses three inputs: $y = x_1^{\frac{1}{5}} x_2^{\frac{1}{5}} x_3^{\frac{1}{5}}$. Suppose $w_1 = w_2 = w_3 = 1$.

   a. (2 points) What are the conditional factor demands $x_1^*(y)$, $x_2^*(y)$, and $x_3^*(y)$.

   b. (1 point) Find the long-run cost function $C(y)$.

   c. (1 point) Find the long-run supply curve $y^*(p)$.

5. (3 points) Let the firm's production function be given by $y = x_1 + x_2$. Suppose $w_1 = 2$ and $w_2 = 1$.

   a. (1 point) Show that this production function exhibits constant returns to scale.

   b. (2 points) Derive the conditional factor demands and use them to find the long-run cost function for this firm.

6. (2 points) XYZ Manufacturing Company specializes in the production of electronic devices, including smartphones and tablets. Their manufacturing process combines capital resources (machinery and equipment) with labor to create these devices. In their pursuit of staying competitive, they continuously seek ways to reduce production costs while maintaining a consistent level of output. Their production output is determined by $y = KL$, where K represents capital and L represents labor. Labor is priced at \$10 per unit, while capital is priced at \$2 per unit. XYZ Manufacturing Company has been diligently monitoring the marginal product of both labor and capital.

   a. (1 point) Is the company effectively minimizing their production costs at the current level of production, given that the marginal product of labor is 9 and the marginal product of capital is 5?

   b. (1 point) If not, how much capital and labor should the company uses to minimize costs while maintaining the current output level?