ECON 251: Intermediate Microeconomics
Department of Economics, Lafayette College Spring 2024
Instructor: Prof. Sayorn Chin (he/him/his)

Problem Set 4

Due: 21 Feb (Physical Copy in Class)

Instruction

Work in teams of 2-3 people, provide your responses typed in LaTeX. Handwritten submissions will not be accepted. The primary purpose of the problem sets is to give you experience thinking and working through economic problems. Getting the right answer is much less important than understanding the right answer and how it was derived. Accordingly, problem sets are graded on a combination of effort and accuracy. Your investment or lack of investment in these assignments will determine your success in the course as problem set investment is strongly correlated with exam performance.

1. (3 points) Consider the utility function $u(x_1, x_2) = 2x_1^{1/2} + x_2$ with income M, price of x_1 p_1, and price of x_2 p_2.

   a. (2 points) Show that the optimal demand functions are $(x_1^*, x_2^*) = \left(\frac{p_2^2}{p_1^2}, \frac{p_1 M - p_2^2}{p_2 p_1}\right)$.

   b. (1 point) Find the optimal consumption bundle, assuming $M = \$100$, $p_1 = \$3$, and $p_2 = \$1$. Illustrate this bundle on a graph.

2. (4 points) Consider the utility function $u(x_1, x_2) = x_1 x_2$. Suppose the initial situation is given by $p_1 = p_2 = 1$ and $M = 10$.

   a. (2 points) If the price of good 1 rises to 2.50, show that the total effect on the consumer's demand for good 1 equals -3.

   b. (2) Show that the total effect can be decomposed into a substitution effect of $\sqrt{10} - 5$, and an income effect of $2 - \sqrt{10}$.

3. (3 points) Suppose there are 12,000 consumers in a country called Cakeland. All of them have the same preferences, represented by the utility function $u(x_{1i}, x_{2i}) = x_{1i} x_{2i}$, where x_{1i} is consumer i's consumption of all goods except cakes and x_{2i} represents i's consumption of cakes. Suppose there are 2,000 consumers with incomes of \$200 each, 3,000 consumers with incomes of \$160 each, and 7,000 consumers with income of \$60 each.

   a. (2 points) Derive the market demand for cakes.

   b. (1 point) Plot the inverse market demand curve for cakes.

4. (7 points) Esther lives in Easton, Pennsylvania. She finds satisfaction in baking delicious cakes and reading captivating novels. Her utility function is expressed as:

$$u(c, n) = c^3 n^5$$

Here, c represents the number of cakes baked, and n represents the number of novels read.

a. (1 point) Calculate Esther's MRS as a function of (c, n).

b. (1.5 points) Sketch Esther's indifference curve that goes through the point $(c, n) = (1, 1)$. Calculate the MRS at this point and represent it on the graph. Identify which good holds higher value at this specific point, cakes (c) or novels (n).

c. (2 points) Using the Lagrange function method, derive Esther's demand functions for cakes and novels.

d. (2.5 points) To address the rising obesity rates in the Easton area, the governor of Pennsylvania has introduced an *ad valorem* sales tax, denoted as τ, on cakes. This tax means that when purchasing cakes at a price of p_c, Esther must now pay $(1 + \tau)p_c$ for them.

   i. (0.5 points) Write down Esther's new budget constraint equation after this new regulation.

   ii. (1 point) Using the tangency condition $MRS = \frac{MU_c}{MU_n} = \frac{p_c}{p_n}$ where p_n is the price of novels, derive her new demand functions for c^* and n^*.

   iii. (1 point) Draw the demand curve before the tax and after the tax in the same graph for cakes (you don't need to assume any particular values for income, prices, and sales tax, it is enough to provide a qualitative graph). Briefly explain the economic intuition.

5. (3 points) Consider the demand function $x_1(p_1, p_2, M) = \frac{M}{p_1+p_2}$.

   a. (1 point) Derive the elasticity of the demand for good 1 with respect to its price as a function of all the variables; that is, find the function $\epsilon_{x_1,p_1}(p_1, p_2, M)$. Provide an explanation.

   b. (1 point) Derive the elasticity of the demand for good 1 with respect to income as a function of all the variables; that is, find the function $\epsilon_{x_1,M}(p_1, p_2, M)$. Provide an explanation.

   c. (1 point) Derive the cross elasticity of the demand for good 1 as a function of all the variables; that is, find the function $\epsilon_{x_1,p_2}(p_1, p_2, M)$. Provide an explanation.